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Thermomechanical peeling in multilayer beams and plates—a solution from first principles

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Abstract

The purpose of this paper is to present a solution for the peeling moment arising from the peeling stress distribution in any interface in a multilayer beam or plate. The solution is implemented for three- and four- layer beams; it is shown that it can readily be implemented for any desired number of layers. The solution is derived from first principles, and is evolved from the well known [Timoshenko, A. 1925. Analysis of bi-metal thermostats. Journal of the Optical Society of America, 11, 233–255] bimetal thermostat analysis. A physical interpretation of the factors that make up the peeling moment is given, enabling quick identification of how any layer property may be changed in order to resist delamination at any interface of interest. The concept of moments being transferred across the interface to cause equal radii of curvature is helpful in understanding the factors that influence the magnitude and direction of the peeling moment at any interface. This analytical method applies equally to multilayer stack-ups cured simultaneously at a common temperature, and to products such as integrated circuit wafers where the layers are formed sequentially at different temperatures.

Keywords: Delamination; Stress; Layer; Peel; Beam; Plate

1. Background

Consider a beam (or plate) built up from two or more layers bonded together at each interface. The materials of the layers may have different coefficients of thermal expansion (CTE). The bond at any interface may be formed at a temperature different from the temperature at which any other interface is bonded. The beam may be used at a temperature different from any bonding temperature.

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Thermomechanical stresses will arise in the beam causing the beam to bend as a result of the different CTEs of adjoining layers. Equal and opposite shear stresses occur at the adjoining surfaces at an interface in order to achieve identical displacements there. One side of the interface is subject to axial compression and the other to axial tension. Reaction forces arise in the axial direction within the stack-ups on either side of the interface, one compressive and one tensile in response to the enforced strains in the axial direction. The net effect of the reaction force in a stack-up lies along the neutral axis of the stack-up.

The shear force at the interface and the net reaction force within the layer are offset, and these create a bending moment on the layer, and corresponding bending stresses. Generally these moments alone do not cause identical radii of curvature of the two layers, so additional peeling stresses arise at each interface. Timoshenko (1925) provided a solution for the axial and bending stresses in both layers of a bimaterial beam, for the total shear force generated at the interface, and for the radius of curvature. However, his solution did not deal with the peeling stress except to observe that both shear and peel were concentrated near the free ends.

Hess (1969a) developed an eigenfunction solution for end-loading in a two-layer beam; the solution was truncated to 30 terms. He superimposed this on Timoshenko's solution for the thermal loading in a bimaterial beam (Hess, 1969b). This enabled the distribution of shear and peeling stresses to be determined; both were concentrated near the free end. The shear stress was equipollent, and decayed to a negligible value at the free end. On the other hand the peeling stress always changed sign in that region and terminated with its maximum value at the free end. Hess found that where both the modulus of elasticity and thickness of one layer were greater than those of the other, the sign of the peeling stress could be determined by a simple formula; however in the general case it was unpredictable.

Chen and Nelson (1979) adapted lap joint theories to examine various joints bonded with compliant adhesives under temperature change in electronic packaging. One such joint was equivalent to a simply supported bimaterial beam; in this the peeling stress reduced to a sixth order differential equation. As in the Hess solution, the peeling stress changed sign close to the free end; however the shear stress continued to increase to its maximum value at the free end rather than reduce to zero to meet equilibrium conditions.

Various simpler approximate solutions have been developed over the years for the peeling stress in bimaterial beams. Many are adapted from Timoshenko's solution. Suhir (1986) added the concept of interfacial compliance and suggested as an approximation that a deviation of longitudinal displacement from that predicted by Timoshenko was proportional to the local interfacial shear stress. The equations were relatively simple to resolve but the resultant interfacial shear and peeling stresses differed from results from FEM and Hess's method (Eischen et al., 1990). Ru (2002) adapted Suhir's approach by suggesting that the deviation of the longitudinal displacement from the Timoshenko model was dependent not only on the local interfacial shear stress, but also on its second derivative; his approximation gave fourth order derivatives in which the constants were easily computed, and the solution was consistent in form to the results as given by Eischen.

The approximate approaches were extended to include multilayer beams in which all layers were bonded at a common temperature. Grimado (1978) examined multilayer beams by focussing on one interface and using a set of effective averaged properties for the portion of the stack above the interface and another set for the portion below; he used a compliant adhesive to remove the free edge singularities, and developed a single sixth-order differential equation to determine the axial forces in the beam, and from that the remaining stresses in the interface in question. Pan and Pao (1990) presented a set of first order equations based on conventional thin plate theory to determine displacement and strain in multilayer plates; shear and peeling were not included. Pao and Eisele (1991) extended the approach of Suhir (1986) to multiple layers to generate a coupled series of linear second order differential equations; the solution did not deliver the expected change of sign of the peeling stress near the free ends. An exact solution for bending and axial stress in multilayer beams was developed where a relatively thick substrate bore thin film layers, also bonded at a common temperature (Hsueh, 2002); again peeling and shear were not addressed.

Moore and Jarvis (2003) presented a simple beam theory solution for peeling in a bimaterial beam under a uniform change in temperature. This enabled prediction of the sign of the peeling stress at the free end; it applied with certainty for all cases irrespective of layer thickness and elasticities. They extended Timoshenko's work to get a simple formulation for the moment arising from the naturally occurring reversal of the peeling stress distribution in the interface. The sign of the peeling moment could be determined by inspection, being dependent only on the thickness and elasticity of each layer and the sign of the radius of curvature (which itself could be determined by inspection). A positive peeling moment gave a tensile peeling stress at the free edge, i.e. promoting peeling; a negative peeling moment gave a compressive peeling stress at the free edge, i.e. resisting peeling and delamination.

In the present paper this work is extended to determine the peeling moment in each interface in a multilayer beam, including the bonding of each interface at a different temperature. The solution can be readily implemented in a spreadsheet, permitting fast analysis of the effect of varying the properties of any layer or bonding condition.

In Section 2 the formulation of the peeling moment in the bimaterial beam is restated in a manner that is easily extended to additional layers. This formulation is extended to any interface in a multilayer beam of four layers (Section 3). In Section 4 the proof is developed for the three-layer beam, while in Section 5 the proof is demonstrated for all interfaces in a four-layer beam. The remaining sections contain validation of the results by the finite element method (FEM), a brief discussion on the application of the solution, and conclusions from the work.

2. Transfer of peeling moment across the interface

In this section a general interpretation of the peeling moment is developed, in which a simple description of the moment at the interface in a bimaterial beam is presented.

Consider the bimaterial beam of Timoshenko; its cross-section is shown in Fig. 1. Let E_i, h_i and α_i be the elastic modulus, thickness and CTE where $i = 1$ or 2. Let T_1 be the temperature at which the interface between layers 1 and 2 are bonded.

A change in temperature causes reaction stresses in each layer in response to the enforced identical axial displacement at the interface; remote from the ends of the beam the summation of stresses in a cross-section of the layer is equivalent to a single reaction force acting through the neutral axis (NA_1 or NA_2) of the respective layer. The distances from the neutral axes of each layer to the interface are $h_1/2$ and $h_2/2$, respectively. The sum of these distances gives the moment arm (*arm*) between the reaction forces. The fractions of the moment arm that lie in the upper layer U and lower layer L are

$$U = \frac{h_1/2}{h_1/2 + h_2/2} \quad \text{and} \quad L = \frac{h_2/2}{h_1/2 + h_2/2} \tag{1}$$

while the sum of these fractions is unity.

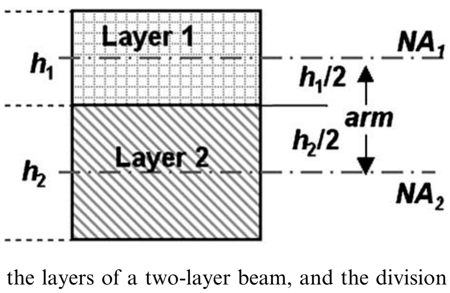

Fig. 1. The interface between the layers of a two-layer beam, and the division of the moment arm across it.

The peeling moment acting on the lower layer from Moore and Jarvis (2003) is

$$M_{\mathrm{p}} = \frac{h_1 E_2 I_2 - h_2 E_1 I_1}{(h_1 + h_2)\rho} \tag{2}$$

where ρ is the radius of curvature of the beam. This can be restated as

$$M_{\mathrm{p}} = U\frac{E_2 I_2}{\rho} - L\frac{E_1 I_1}{\rho} \tag{3}$$

But $E_2 I_2/\rho$ is the moment required to bend the lower layer to radius ρ, and similarly $E_1 I_1/\rho$ is the moment required to bend the upper layer. Thus the peeling moment at the interface is made up of the upper fraction U times the moment to bend the lower layer, less the lower fraction L times the moment required to bend the upper layer.

The physical explanation for this constitution in Eq. (3) is developed in Appendix A. There it is shown that the peeling moment in a bimaterial beam can be considered as the sum of the moments transferred across the interface into and out of the lower layer; these are the contribution of the reaction in the upper layer to the bending in the lower layer less the contribution of the reaction in the lower layer to bending in the upper layer.

3. Proposition: general transfer of moments by the peeling stress

When a third layer is attached, an additional phenomenon occurs. There is a pair of layers forming a stack-up on one side of the interface and a single layer on the other. The stack-up (say layers 1 and 2) is bonded at a particular temperature and forms a bimaterial beam. At the temperature at which the second interface is created (bonding of layer 3 to layer 2) the bimaterial beam would be bent to radius of curvature ρ_{12}. However for bonding the second interface the bimaterial beam is required to be straight; for this a temporary external moment of $-M_{12}$ is applied. A corresponding reaction moment arises within the bimaterial beam to oppose the temporary straightening moment. The reaction moment M_{12} at the temperature T_2 of bonding of the layer 2 to 3 interface is

$$M_{12} = \frac{E_{12} I_{12}}{\rho_{12}} \tag{4}$$

where E_{12} is the average modulus of elasticity weighted by the thicknesses of the layers, I_{12} is the moment of inertia of the bimaterial beam with reference to its neutral axis NA_{12} (again weighted by the thicknesses of the layers), and ρ_{12} is the radius of curvature of the free bimaterial beam at the temperature of bonding of the second interface. The computation of average properties of stack-ups is done in accordance with the methods used by Boley and Testa (1969) and Grimado (1978).

This proposition asserts that moments which arise on one side of the interface are transferred in part to the other side by means of the peeling moment. The fraction of each moment that is transferred is the same as the fraction of the receiving side (U or L) in the total moment arm between the neutral axes of the stack-ups on each side of the interface. The peeling moment is the algebraic sum of these transferred fractions.

For example it can be stated that in a four-layer beam such fractional distributions of moments arises. Taking the central interface, and counting layers from top to bottom, the division of the moment arm across the interface between layers 2 and 3 is illustrated in Fig. 2. Let the radius of curvature of the four-layer beam be ρ_4.

There can be two simultaneous forms of moment in a stack-up. One moment is due to the action of the interfacial shear on the layer, and the other moment is due to the bimaterial effect where the stack-up

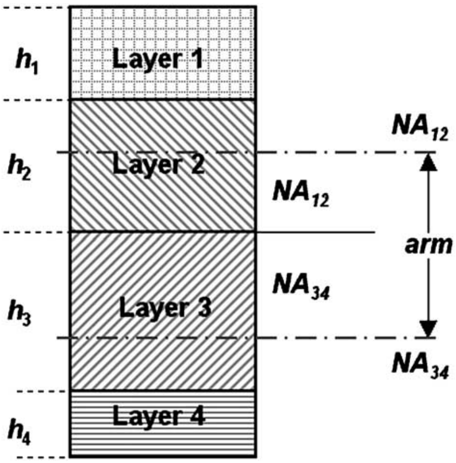

Fig. 2. The interface between layers 2 and 3 of a four-layer beam, and the division of the moment arm across it.

comprises of more than one layer (of course when the stack-up consists of only one layer this latter moment is zero).

The bimaterial beam consisting of the lower pair of layers 3 and 4 has a reaction moment M_{34} arising within it when it is straightened at the temperature T_2 for bonding of interface the between layers 2 and 3:

$$M_{34} = \frac{E_{34}I_{34}}{\rho_{34}} \tag{5}$$

where E, I and ρ are as defined following Eq. (4) except that they relate to layers 3 and 4 instead of layers 1 and 2.

The peeling moment in an interface is the algebraic sum of the portions of the moments that are transferred across the interface. Its general arrangement is set out in Eq. (6). Moments acting *on* the lower side of the interface are positive, while moments which are the *action from* that side of the interface are negative. This particular equation gives the peeling moment M_p acting on the lower stack-up in the middle interface of a four-layer beam as shown in Fig. 2. The beam is bent to radius of curvature ρ_4.

$$M_{P24} = U \cdot \frac{E_{34}I_{34}}{\rho_4} - L \cdot \frac{E_{12}I_{12}}{\rho_4} + L \cdot M_{12} - U \cdot M_{34} \tag{6}$$

This equation may be interpreted as meaning that all moments relating to the stack-up on one side of an interface are distributed across the interface to the other side in proportion to the distances of the neutral axes of each side from the interface. The explanation which follows is an interpretation of the role of the component parts of this equation.

The first item on the right-hand side of the equation is the contribution from the action of shear on the upper stack-up to bending the lower stack-up; it is acting on the lower stack-up and for this reason its effect is positive.

The second item is the reciprocal contribution from the action of shear on the lower stack-up to bending the upper stack-up; it is an action from the lower stack-up and for this reason its effect is negative.

The moment M_{12} relates to an independent bimaterial stack-up of layers 1 and 2 as described in Eq. (4). This beam would take up a curvature ρ_{12} under the temperature change. The third item is the portion of the moment M_{12} that contributes to bend the lower stack-up. It is acting on the lower stack-up and is thus a positive fraction.

The final item is the portion of the corresponding moment M_{34} within the lower stack-up that contributes to bending the upper stack-up; it is an action from the lower stack-up and is a negative fraction. M_{34} is determined in the same way as M_{12}.

The equations for the peeling moments in the other interfaces are identical in pattern to Eq. (6).

The proposition can be extended in the same way to determine the value of the peeling moment at any interface in a beam with any desired number of layers. In the next section the basis of the proposition is proven.

4. Proof of the proposition—three layers

Consider the build-up of a multilayer beam with the uppermost layer as layer 1 and succeeding layers numbered incrementally. Let E_i, h_i and α_i be the elastic modulus, thickness and CTE of a single layer i. Let T_i be the temperature at which the interface between layers i and $i+1$ are bonded. Let the centroids of combinations of layers be calculated with reference to the top surface of the top layer. A sketch of the first two layers is given in Fig. 3.

The bending in the bimaterial beam comprising of layers 1 and 2 may be computed following Timoshenko (1925) and the peeling moment may be computed following Moore and Jarvis (2003). This beam is bent with radius ρ_2.

To analyse the tri-layer beam it is necessary to treat a pair of layers as an equivalent single beam with a built-in or residual moment. It is convenient to combine layers 1 and 2, although layers 2 and 3 may be combined instead. It is not necessary to combine the layers in the sequence in which they are bonded. The arrangement is shown in Fig. 4.

The thickness of the composite layer is h_{12}, the sum of the thicknesses h_1 and h_2. The modulus of elasticity is the average modulus of the two layers weighted by the layer thicknesses:

$$E_{12} = \frac{E_1 h_1 + E_2 h_2}{h_1 + h_2} \tag{7}$$

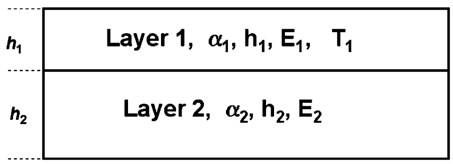

Fig. 3. Properties of layers 1 and 2.

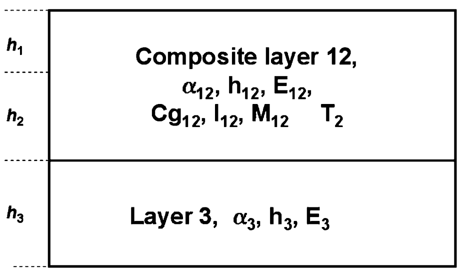

Fig. 4. Properties of composite beam 1 and 2 and layer 3.

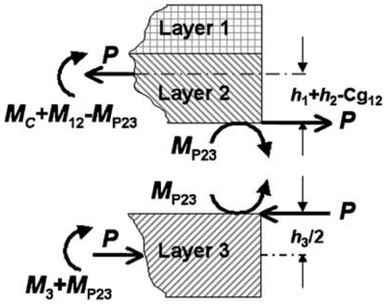

Fig. 5. Free-body diagrams of the two sides of the new interface.

The neutral axis of the composite beam lies at the centroid of the beam, Cg_{12}. The method of calculating the centroid is as used following Fig. 5 of Timoshenko (1925), but expanded for clarity:

$$Cg_{12} = \frac{E_1 h_1^2/2 + E_2 h_2^2/2 + E_2 h_1 h_2}{E_1 h_1 + E_2 h_2} \tag{8}$$

The CTE of the composite layer is found in the same way

$$\alpha_{12} = \frac{\alpha_1 E_1 h_1 + \alpha_2 E_2 h_2}{E_1 h_1 + E_2 h_2} \tag{9}$$

The moment of inertia I_{12} of the composite layer about its neutral axis (at Cg_{12}) is

$$I_{12} = \frac{1}{E_{12}}[E_1 I_1 + E_1 h_1 (h_1/2 - Cg_{12})^2 + E_2 I_2 + E_2 h_2 (h_1 + h_2/2 - Cg_{12})^2] \tag{10}$$

The reaction moment M_{12} within this composite beam when it is straightened for bonding to layer 3 at temperature T_2 is given previously in Eq. (4).

Now consider the tri-layer beam, as shown in Fig. 5. The shear stress in the interface acting on a layer is balanced by the reaction force P within the layer. Let M_3 be the moment arising from the shear force of magnitude P acting on layer 3, and let M_C be the moment arising from the shear force P acting on the stack-up of layers 1 and 2. The moment arms are $h_3/2$ and $h_1 + h_2 - Cg_{12}$, respectively.

A peeling moment arises in the new interface of the three-layer beam, between the stack-up of layers 1 and 2, and layer 3. It is designated M_{P23}. Free-body diagrams of the forces and moments on the two sides of the interface are shown in Fig. 5.

There are two moments acting to bend layer 3; these are $M_3 = Ph_3/2$ and the peeling moment, M_{P23}. There are three moments acting to bend the composite layer; these are $M_C = P(h_1 + h_2 - Cg_{12})$, together with its reaction moment M_{12} and the peeling moment of opposite sign $-M_{P23}$. The sum of all these moments causes the two layers to bend to ρ_3; this sum is $P(h_1 + h_2 - Cg_{12} + h_3/2) + M_{12}$. Then rearranging to get an expression for the reaction force P:

$$P = \frac{\frac{E_{12}I_{12}+E_3 I_3}{\rho_3} - M_{12}}{h_1 + h_2 - Cg_{12} + h_3/2} \tag{11}$$

Using Timoshenko's approach of compatibility of displacements in the interface between layers 2 and 3 we can write for the condition at any temperature T where T_2 is the temperature of bonding of the interface 2–3:

$$\frac{h_1 + h_2 + h_3/2}{\rho_3} + \frac{\frac{E_{12}I_{12}+E_3 I_3}{\rho_3} - M_{12}}{h_1 + h_2 - Cg_{12} + h_3/2} \cdot \left(\frac{1}{E_{12}(h_1 + h_2)} + \frac{1}{E_3 h_3}\right) = (\alpha_3 - \alpha_{12}) \cdot (T - T_2) \tag{12}$$

From this the expression for $1/\rho_3$ is obtained:

$$\frac{1}{\rho_3} = \frac{(h_1 + h_2 - Cg_{12} + h_3/2)(\alpha_3 - \alpha_{12}) \cdot (T - T_2) + M_{12} \cdot \left(\frac{1}{E_{12}(h_1+h_2)} + \frac{1}{E_3 h_3}\right)}{(h_1 + h_2 - Cg_{12} + h_3/2)^2 + (E_{12}I_{12} + E_3 I_3) \cdot \left(\frac{1}{E_{12}(h_1+h_2)} + \frac{1}{E_3 h_3}\right)} \tag{13}$$

This Eq. (13) can be rearranged as

$$\frac{1}{\rho_3} = \frac{(\alpha_3 - \alpha_{12}) \cdot (T - T_2)}{(h_1 + h_2 - Cg_{12} + h_3/2) + \frac{(E_{12}I_{12}+E_3 I_3)}{(h_1+h_2-Cg_{12}+h_3/2)} \cdot \left(\frac{1}{E_{12}(h_1+h_2)} + \frac{1}{E_3 h_3}\right)} + \frac{M_{12}}{\frac{(h_1+h_2-Cg_{12}+h_3/2)^2}{\left(\frac{1}{E_{12}(h_1+h_2)}+\frac{1}{E_3 h_3}\right)} + (E_{12}I_{12} + E_3 I_3)} \tag{14}$$

The first part of the right-hand side of this equation is identical to the Timoshenko formulation for a bimaterial beam consisting of an upper layer of thickness $h_1 + h_2$ (with the weighted average properties of the composite beam from layers 1 and 2) and layer 3 as the lower layer. The second part is the bending of all three the layers arising from the reaction moment M_{12} within the composite beam.

The effects of the Timoshenko bending and the reaction moments remain separated throughout the analyses below.

As shown in Fig. 5, the moments acting to bend layer 3 are $Ph_3/2$ and M_{P23}, where M_{P23} is the peeling moment in second interface of the three-layer beam:

$$\frac{1}{\rho_3} = \frac{M_3 + M_{P23}}{E_3 I_3} \tag{15}$$

From which, using $M_3 = Ph_3/2$ and substituting from Eq. (11) for P:

$$M_{P23} = \frac{E_3 I_3}{\rho_3} - \frac{h_3/2}{h_1 + h_2 - Cg_{12} + h_3/2}\left[\frac{E_{12}I_{12} + E_3 I_3}{\rho_3} - M_{12}\right] \tag{16}$$

But the multiplier outside the square brackets is the lower fraction of the moment arm L, computed in the same manner as in Eq. (6). Remembering that $U + L = 1$, Eq. (16) can be rearranged as

$$M_{P23} = U\frac{E_3 I_3}{\rho_3} - L\frac{E_{12}I_{12}}{\rho_3} + L \cdot M_{12} \tag{17}$$

Thus M_{P23} is the sum of the fractions of the moments transferred across the interface, as proposed in Section 2. As mentioned above, the effects of the simple bimaterial bending and the reaction moment M_{12} are clearly separated.

The peeling moment in the upper interface of the three-layer beam is labelled M_{P13}. It can be found by following the same sequence of analysis as in this section with the stack-up of layers reversed.

5. Four layers and beyond

In a four-layer beam, the logic of the previous section is followed in order to obtain M_{P34} and in the same way, M_{P14}; these are the peeling moments in the two outer interfaces. The analysis starts by treating the tri-layer beam examined in Section 4 as a composite beam with properties shown in Fig. 6. This is then attached to layer 4, and the analysis continues exactly as in Section 4.

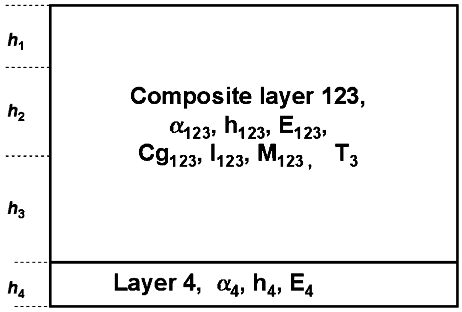

Fig. 6. Properties of composite beams 1 and 2 and 3, and layer 4.

It will be found that the format of M_{P34} is very similar to that of M_{P23}:

$$M_{P34} = U\frac{E_4 I_4}{\rho_4} - L\frac{E_{13} I_{13}}{\rho_4} + L \cdot M_{13} \tag{18}$$

For the peeling moment at the central interface M_{P24}, the equation is also readily obtained. The arrangement of the layers comprising this interface is shown in Fig. 7.

The moments acting on the lower stack-up (made up of layers 3 and 4) to cause it to bend to ρ_4 at the bonding temperature of the second interface T_2 are

$$\frac{E_{34} I_{34}}{\rho_4} = P \cdot (Cg_{34} - h_1 - h_2) + M_{34} + M_{P24} \tag{19}$$

where P is the reaction force to shear in the second interface of this four-layer beam, and M_{34} is the moment within the stack-up of layers 3 and 4 when it is straightened for bonding at temperature T_2.

The moments acting on the upper stack-up (layers 1 and 2) to cause it to bend to ρ_4 are

$$\frac{E_{12} I_{12}}{\rho_4} = P \cdot (h_1 + h_2 - Cg_{12}) + M_{12} - M_{P24} \tag{20}$$

Adding these equations eliminates M_{P24} and yields:

$$P = \frac{\frac{E_{34} I_{34}}{\rho_4} + \frac{E_{12} I_{12}}{\rho_4} - M_{12} - M_{34}}{(Cg_{34} - Cg_{12})} \tag{21}$$

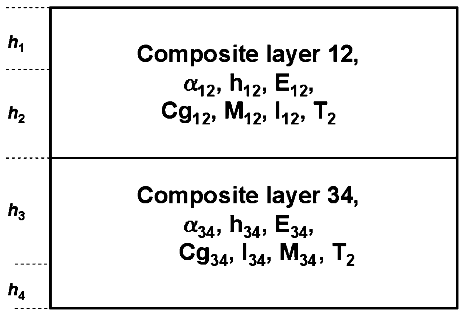

Fig. 7. Properties of composite beams 1 and 2 and 3 and 4.

Substituting this into Eq. (19) and taking M_{P24} to one side gives:

$$M_{P24} = \frac{E_{34}I_{34}}{\rho_4} - \frac{(Cg_{34} - h_1 - h_2)}{(Cg_{34} - Cg_{12})} \cdot \left[\frac{E_{34}I_{34}}{\rho_4} + \frac{E_{12}I_{12}}{\rho_4} - M_{12} - M_{34}\right] - M_{34} \tag{22}$$

Note that the multiplier before the expression in square brackets is the fraction L of the moment arm between the upper and lower neutral axes with respect to the second interface of the four-layer beam. Since $U + L = 1$, Eq. (22) can be rearranged as

$$M_{P24} = U \cdot \frac{E_{34}I_{34}}{\rho_4} - L \cdot \frac{E_{12}I_{12}}{\rho_4} + L \cdot M_{12} - U \cdot M_{34} \tag{23}$$

Note the similarity to Eq. (18) with the introduction of the second reaction moment, M_{34}; the upper component of this moment is transferred across the interface. This demonstrates again the validity of the proposition set out in Section 3.

It has been directly shown for both an exterior interface (i.e. the third interface of the four-layer beam) and an interior interface (i.e. the second interface of the four-layer beam) that the peeling moment is composed of the algebraic sum of the moments transferred across the interface. It is clear that the peeling moments at the interfaces in a beam with additional layers may be computed in the same fashion. As the analytical method is iterative, it lends itself to easy implementation in a spreadsheet.

6. Example results

The analyses described above were implemented in MathCAD for a sample beam comprising of four layers. The properties of the beam are set out in Table 1 with layer 1 on the top of the stack-up and layer 4 on the under-side. A beam half-length ($W/2$) of 50 mm, and a uniform temperature change of −240 °C were selected.

These properties were used to find the peeling moments and interfacial shear forces in the three interfaces of the beam. The results are given in Table 2 for the actions on the lower side of the interface. The actions on the upper side of the interface are of opposite sign.

The geometry and materials of this sample beam were chosen in order to ensure a non-graduated stack-up of CTEs, thicknesses and moduli of elasticity. They were not selected with any real-life application in mind, but rather to deliver an example with positive peeling moments in all interfaces.

Table 1
Layer properties for a sample four-layer beam

Layer no	Thickness (m)	Coefficient of thermal expansion/°C	Elastic modulus (N/m^2)
1	8.0E−4	29.0E−6	90E+9
2	5.0E−4	12.4E−6	20E+9
3	6.0E−4	2.0E−6	100E+9
4	4.0E−4	15.0E−6	180E+9

Table 2
Peeling moment and interfacial shear forces acting on lower surface of each interface in sample four-layer beam

Interface	1–2	2–3	3–4
Peeling Moment (Nm)	+17.82	+47.05	+24.70
Shear Force (N)	−67,080	−51,800	+112,200

Table 3
Y-displacements (in metres) at all interfaces by three methods

Interface	1–2	2–3	3–4
Beam theory analysis FEM	2.932E−3	2.932E−3	2.932E−3
bonded beam FEM + theory	2.930E−3	2.930E−3	2.930E−3
Upper stack-up	2.938E−3	2.924E−3	2.931E−3
Lower stack-up	2.931E−3	2.938E−3	2.942E−3

7. Results of theory demonstrated by FEM

These beam structures were analysed with finite elements using ANSYS 6.1. The FEM model consisted of a four-layer beam with all layers bonded, plus a separate replication of each layer as an independent beam. This followed the method used by Moore and Jarvis (2003). It was adapted to permit one or two layers to be omitted if desired. Thus it provided for applying to the relevant layer or partial stack-up the shear force and peeling moment as determined from the beam theory analysis. This allowed demonstration that the *Y*-displacements as computed by beam theory ($Uy = W^2/2\rho$) were also found by the FEM approaches, both in the bonded beam and the independent layers estimations.

In the FEM structure the half-beam to one side of a vertical line of symmetry was modelled; all layers were identical with 456 four-noded elements of PLANE42 type for each layer, in plane stress. The greater portion of the elements in each layer was concentrated in the 10% of the length close to the free edge. In the FEM the reference temperature for each layer was set at zero, and a thermal load of −240 °C was applied. The resulting *Y*-displacements at each interface are set out in Table 3. This table compares the results from the beam theory method, the FEM of a bonded beam, and the analysis by FEM of a single layer or partial stack-up when the shear force and peeling moment results from the beam theory method are applied to it.

For the separate stack-up analyses the shear force was applied to the node at the free edge of the interface surface of the stack-up, and the moment was applied as a pair of forces acting vertically through the same node and the next neighbouring node—located 0.00025 m inboard of that. The temperature change of −240 °C was applied.

Convergence of the FE model was validated by rerunning all analyses using PLANE82 eight-noded elements. This resulted in 11,521 nodes, instead of 2964. All values in Table 3 were unchanged to four significant digits in the new analysis.

There is clearly a concurrence of all *Y*-displacement values irrespective of how they were determined. As expected, the results from the beam theory method are identical to the results from the four-layer FEM within less than 0.1%. The demonstration of the theory lies in the results obtained by applying the shear force and peeling moments from beam theory to the partial stack-ups in FEM. In the worst case these had a *Y*-displacement error of less than 0.3% of the beam theory displacement.

8. Discussion

The objective of the analysis was to find from beam theory an expression for the peeling moment at any interface in a multilayer beam or plate. This has been achieved, and has the advantage that the expression has a physical explanation that is intuitively acceptable. The concept of moments being transferred across the interface to cause equal radii of curvature is helpful in understanding the factors that influence the magnitude and direction of the peeling moment at any interface.

In Eq. (14) the bending due to differences in CTE across the interface, was separated from the bending due to the reaction moments in the upper and lower stack-ups. This separation enabled the consequent separation of the components of the peeling moment. The composition of the equation for the peeling moment is clearly quite similar at any interface. The partitioning of the moment arm into the fractions above and below the interface is an essential feature of this composition.

By separating the parts of the peeling moment in this way, the relative contributions of the reaction moments in the upper and lower stack-ups, and those of the axial reactions within the stack-ups become clear. With this understanding of the components of the peeling moment, the probable direction of the effect of a change of thickness or material property in an individual layer on the peeling stress at the free edge at an interface can be estimated, irrespective of whether the layer adjoins the interface or not.

The material properties and thicknesses selected for the worked example were not drawn from reality. Instead they were chosen to allow a variety of conditions to be examined. All materials were assumed to be isotropic, and all properties were assumed to be constant with temperature. Clearly this is not the case in reality; however, the use of material properties based on the secant of the property/temperature curve is an acceptable substitute for the purposes of first-order estimates. The Y-displacement data for the point at the free end of the interface in question is a useful benchmark. The data from FEM of the independent layers concurs to within 0.3% with the data computed from beam theory as set out in this paper.

The solution is developed from first principles. Accordingly it can be used in all multilayer situations. These include clad plates and beams, multilayer assemblies such as the packaging of integrated circuits, multilayer protective coatings, and the conductor/dielectric stack-up on integrated circuits.

9. Conclusions

A solution has been developed from first principles for the peeling moment in a multilayer beam at a uniform final temperature. The solution is presented in a form that is easy to conceptualise. This allows a ready prediction of the probable effect on the peeling moment at any interface, of varying the thickness or property of any layer.

The solution is generic, and is almost as easy to implement for a complex multilayer beam as it is for a tri-layer beam. The concepts remain identical irrespective of the complexity of the beam.

Appendix A

The purpose of this appendix is to develop a physical explanation for the content of Eq. (3), viz.

$$M_\mathrm{p} = U\frac{E_2 I_2}{\rho} - L\frac{E_1 I_1}{\rho} \tag{A.1}$$

Timoshenko (1925) stated that shear arose between the layers close to the ends of the bimaterial beam, and designated the accumulated shear stress at one end of the layer as the force P. Consider, as in Timoshenko, the case where the upper layer has the shorter length after a change in temperature. The effect of the shear action of the upper layer on the lower layer is to compress the axial length of the lower layer; the reaction within the lower layer is a compressive stress which summed over the cross-section of the layer is equivalent to a compressive force P acting through the neutral axis of the lower layer.

There is a corresponding effect on the upper layer from the shear action of the lower layer. It is a tensile force of magnitude P acting through the neutral axis of the upper layer.

Just as the axial stresses within the layers can be replaced by a single force, so also the shear stresses in the upper and lower layers at the interface can be replaced by statically equivalent forces, both of magnitude P. One force acts in the axial direction at the point on the interface at the free edge of the lower layer; this is a compressive force. The other acts as an axially tensile force at the point on the interface at the free edge of the upper layer. Fig. A.1 shows free-body diagrams of the right-hand ends of the upper and lower

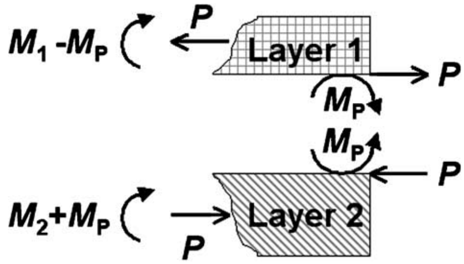

Fig. A.1. Free-body diagrams of right-hand ends of each layer under temperature change.

layers (layers 1 and 2). As used by Timoshenko, M_1 is the reaction moment within layer 1 and M_2 is the reaction moment within layer 2. The direction of the moment arrows in all cases is the sense of the moment where the peeling stress at the free edge is tensile. The value is the magnitude of the moment.

This free-body diagram is extended beyond Timoshenko's work by including for the peeling stresses which arise between the layers close to the free ends. The peeling stresses acting on one layer are self-equilibrating in the region close to the end and may be represented by a single moment. The moment acting on layer 2 is designated M_{p} and its effect is to cause sagging of the layer; that acting on layer 1 has magnitude M_{p}, and its effect is to cause the layer to hog.

By taking advantage of the principle of superposition we can examine separately the effects of the axial forces one pair at a time. Consider first the effect of the tensile forces representing the shear acting on the upper layer. Let these be distinguished from the compressive forces by labelling them P_{T}, and let the resultant moment and bending be labelled M_{PT} and ρ_{T}.

The action of P_{T} on layer 1 has caused the two layers to bend to ρ_{T}. The moment M_{PT} is the only force or moment acting on layer 2. It is transferred across the interface from layer 1 by means of the peeling stresses that arise close to the free edge. Accordingly

$$M_{\mathrm{PT}} = \frac{E_2 I_2}{\rho_{\mathrm{T}}} \tag{A.2}$$

layer 1 also bends to ρ_{T}. The peeling moment M_{PT} promotes hogging and opposes the effect of the axial forces. Thus

$$\frac{E_1 I_1}{\rho_{\mathrm{T}}} = M_{1T} - M_{\mathrm{PT}} = P_{\mathrm{T}} \cdot \frac{h_1}{2} - M_{\mathrm{PT}} = P_{\mathrm{T}} \cdot \frac{h_1}{2} - \frac{E_2 I_2}{\rho_{\mathrm{T}}} \tag{A.3}$$

which can be rearranged to give

$$\frac{1}{\rho_{\mathrm{T}}} = \frac{P_{\mathrm{T}} \cdot h_1/2}{E_1 I_1 + E_2 I_2} \tag{A.4}$$

Substituting this into Eq. (A.2) gives

$$M_{\mathrm{PT}} = \frac{E_2 I_2}{E_1 I_1 + E_2 I_2} \cdot P_{\mathrm{T}} \cdot \frac{h_1}{2} \tag{A.5}$$

From Eq. (1) of the main text, $h_1/2$ is the upper portion of the moment arm $h_1/2 + h_2/2$ between the neutral axes of the two layers; it can be written as $U \cdot (h_1/2 + h_2/2)$.

Eq. (A.5) may be rewritten as

$$M_{\mathrm{PT}} = \frac{E_2 I_2}{E_1 I_1 + E_2 I_2} \cdot P_{\mathrm{T}} \left(\frac{h_1 + h_2}{2} \right) \cdot U \tag{A.6}$$

The magnitude of P_T is P; the expression in brackets is the total moment arm between the reaction forces of the upper and lower layers; the product of these is the moment giving rise to the total bending of the beam to radius of curvature ρ, viz:

$$P_T\left(\frac{h_1+h_2}{2}\right)=\frac{E_1I_1+E_2I_2}{\rho} \tag{A.7}$$

and substituting into Eq. (A.6) gives:

$$M_{PT}=\frac{E_2I_2}{\rho}\cdot U \tag{A.8}$$

M_{PT} is the component of the peeling moment that arises from the tensile effect of the shear stress acting on layer 1. It is equal to the upper fraction U of moment required to bend layer 2 to the full radius of curvature. Even though the tensile stress is acting on layers 1, this moment acts on layer 2 because it is transferred across the interface by the peeling stresses.

Similarly the effects of the compressive force representing the shear acting on the lower layer may also be examined separately. Let the resultant moment and bending be labelled M_{PC} and ρ_C.

The moment M_{PC} is the only force or moment acting on layer 1. It is transferred across the interface from layer 2 by means of the peeling stresses that arise close to the free edge. On layer 2 M_{PC} promotes sagging and supplements the effect of the axial forces. Recognising that M_{PC} on layer 2 acts in the opposite sense to M_{PT} on layer 1 it is clear by comparison with Eq. (A.8) that

$$M_{PC}=-\frac{E_1I_1}{\rho}\cdot L \tag{A.9}$$

M_{PC} is the component of the peeling moment that arises from the compressive effect of the shear stress acting on layer 2. It is equal to the lower fraction L of the moment required to bend layer 1 to the full radius of curvature. The sum of the two components of the peeling moment is

$$M_p=M_{PC}+M_{PT}=U\frac{E_2I_2}{\rho}-L\frac{E_1I_1}{\rho} \tag{A.10}$$

Thus the peeling moment in a bimaterial beam can be considered as the sum of the moments transferred across the interface; these are the contributions of the reaction in the upper layer to the bending in the lower layer less the contributions of the reaction in the lower layer to bending in the upper layer.

This is the physical interpretation of the peeling moment given following Eq. (3).

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